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Harbour oscillations generated by shear flow

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A new mechanism that could be responsible for excitation of long-period oscillations in partially enclosed harbours is discussed. This mechanism is based on the interaction between a shear flow and the harbour-basin natural mode and does not suppose any external exciting forces caused by wind waves, tsunami, etc. The growth rate of harbour oscillations is found in terms of a plane-wave reflection coefficient integrated on the wavenumber spectrum of the oscillating outflow field near the harbour entrance. Analytical considerations for simple shear flows (vortex sheet and jet) show that the growth rate changes its sign depending on the ratio of oscillation frequency to flow speed.

1. Introduction

Strong long-period surface oscillations in harbours cause problems for ships mooring, loading or unloading. Their origin is connected with shallow-water modes inside the harbour basin. The harbour may be considered as a surface-wave resonator with certain oscillation modes and eigenfrequencies. If the harbour entrance is narrow enough the fundamental (Helmholtz) mode with the lowest frequency may be excited. This mode was studied originally by Miles & Munk (1961). The structure and frequency of higher modes are sensitive to the harbour geometrical form (see a comprehensive study in Mei 1989).

But the excitation mechanism of surface oscillations on these modes is still an open question. Some external oscillating forces have been investigated as possible sources of harbour oscillations. These forces may appear owing to low-frequency waves incident on a harbour entrance, for example tsunamis (Miles 1974; Gilmour 1990; Zelt & Raichlen 1990) or tides (Abrol 1990). Another plausible exciting force may originate in long waves by storm wave trains (Okihiro, Guza & Seymour 1993). Free long waves may be generated by the refraction of wave trains on bottom inhomogeneities (Mei & Benmoussa 1984; Liu 1989) or on shear currents (Liu, Dingemans & Kostense 1990) and also by the diffraction by such objects as a vertical cylinder (Zhou & Liu 1987). The nonlinear mechanism of harbour excitation by an induced long wave accompanying a wind-wave train has been also investigated (Bowers 1977; Mei & Agnon 1989; Wu & Liu 1990).

All these studies are based on the assumption that a harbour may be considered simply as a resonant element amplifying external pulsations. Here an attempt is made to explain the occurrence of harbour oscillations without any significant external forces as a result of instability due to the interaction of harbour oscillations with a sea current. It is supposed that the current with a horizontal shear goes along the coast near the harbour entrance. This is a typical situation for many harbours which are located at the ocean shore. The near-coast currents may be stationary, seasonal, tidal or may appear accidentally for some other reason. But even if the current is periodic, for example caused by tides, edge waves or any other wave mode with period low in comparison with the harbour oscillation period, we may consider the flow to a first approximation as stationary and investigate oscillations on a stationary background.

In the following section we consider an analytical model of harbour oscillations interacting with a shear flow near the harbour entrance. A perturbation technique is developed to calculate the reflection coefficient of a surface shallow-water wave propagating in a channel with an open end. In §3 reflection coefficients of a plane surface wave from shear flows are summarized and their relevance to the harbour oscillation growth rate is considered on the basis of the causality principle. Approximate analytical expressions for the growth rate are found in §4 for simple flow velocity profiles, i.e. for a vortex sheet and a thin jet. Finally in §5 conclusions are drawn summarizing the results and discussing their physical relevance.

2. Reflection of a waveguide fundamental mode from the open end of a channel

The shallow-water approximation is typically used for long-scale wave modes in a basin. Two-dimensional fluid flow with velocity V satisfies in this approximation the following equations (Mei 1989):

$$\left.\begin{aligned} \partial V/\partial t+(V\cdot\nabla)\,V+g\nabla h &= 0,\\ \partial h/\partial t+\nabla\cdot(hV) &= 0. \end{aligned}\right\} \tag{1}$$

These equations are the same as the two-dimensional Euler equations for compressible fluid. This well-known analogy allows us to consider shallow-water surface waves in a basin like sound waves in two-dimensional acoustic resonator. The phase speed of shallow-water waves $c=(gH)^{1/2}$ (where H is the undisturbed basin depth) plays the role of sound speed (Landau & Lifshits 1987).

Below we use a simple model of the harbour basin: a straight channel considered as a waveguide for surface waves which is orthogonal to the coast line (see figure 1). The fluid layer depth H is homogeneous both inside the channel and in the sea around. This waveguide may be considered as the end of an open surface-wave rectangular resonator (figure 1*a*) or the throat of a Helmholtz resonator (figure 1*b*). We shall study the waveguide fundamental wave coming from inside the channel and reflecting at the open end. On interacting with a coastal flow near the open end this wave mode may be amplified thus bringing additional wave energy inside the harbour resonator and exciting harbour oscillations. Amplification of harbour oscillations by a shear flow may be considered in terms of the influence of the flow on the waveguide wave reflection coefficient at the open end. We shall here find this coefficient analytically and study a general mechanism of flow interaction with surface oscillations in a semi-closed cavity.

In an area with no stationary flows, perturbations of velocity $\boldsymbol{v}=(u,v)$ and surface elevation η satisfy linearized equations following from (1):

$$\left.\begin{aligned} \partial\boldsymbol{v}/\partial t+g\nabla\eta &= 0,\\ \partial\eta/\partial t+H\nabla\cdot\boldsymbol{v} &= 0. \end{aligned}\right\} \tag{2}$$

For oscillations of amplitude $e^{-i\omega t}$ at a given frequency ω we can derive from (2) the Helmholtz equation for the surface elevation:

$$\Delta\eta+(\omega^2/c^2)\,\eta=0. \tag{3}$$

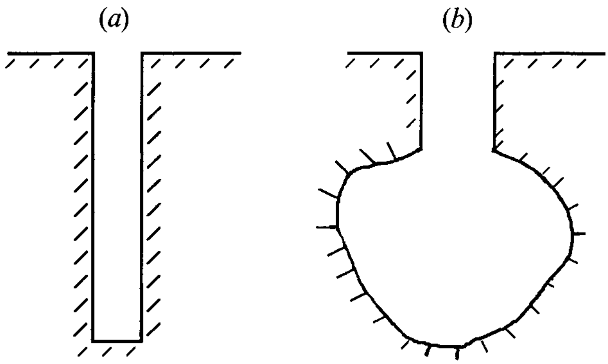

FIGURE 1. Harbour models: (*a*) rectilinear channel, (*b*) Helmholtz surface-wave resonator.

The velocity field is connected with the surface elevation field by the formula

$$v = (g/\mathrm{i}\omega)\,\nabla\eta. \tag{4}$$

Equation (3) must be solved in a semi-closed domain without any flow and with boundary conditions at the domain walls $\boldsymbol{v}\cdot\boldsymbol{n} = 0$ (where the vector $\boldsymbol{n}$ is orthogonal to the wall). Also, some impedance boundary conditions must be imposed at an additional surface closing the domain. Below we consider this boundary-value problem for a particular geometry of the harbour entrance (see figure 2).

If we assume that the waveguide cross-section has a width a small in comparison with the wavelength λ,

$$a \ll \lambda = 2\pi c/\omega, \tag{5}$$

then only the fundamental wave mode can propagate in the waveguide. For this mode the velocity oscillations inside the channel far enough from the open end are directed along the channel axis so that $\boldsymbol{v} = (0, v)$. We can find the velocity and surface elevation fields for the fundamental mode from (3):

$$\left.\begin{aligned} \eta &= \eta_0[\mathrm{e}^{+\mathrm{i}k_0 y} - (R+\Delta R)\,\mathrm{e}^{-\mathrm{i}k_0 y}]\,\mathrm{e}^{-\mathrm{i}\omega t}, \\ v &= v_0[\mathrm{e}^{+\mathrm{i}k_0 y} + (R+\Delta R)\,\mathrm{e}^{-\mathrm{i}k_0 y}]\,\mathrm{e}^{-\mathrm{i}\omega t}, \end{aligned}\right\} \tag{6}$$

where $k_0 = \omega/c$ and velocity and surface elevation amplitudes are connected by the relation

$$\eta_0 = Hv_0/c \tag{7}$$

which follows from (4).

Under condition (5) reflection of the fundamental wave is almost total and the reflection coefficient $R \approx 1$. The oscillating field in a channel is close to a standing wave with a velocity maximum and a node of surface elevation at the open end (Mei 1989).

A deviation from the purely standing wave may appear for at least two different reasons. First, some wave energy radiates from the open end, reducing the wave reflection, so the value of reflection coefficient $|R| < 1$. This effect, however, is of order $(a/\lambda)^2$ and we neglect it below, assuming $R = 1$. Another progressive wave perturbation in a waveguide appears owing to the fundamental wave interaction with the flow outside the channel and is determined by the value of ΔR.

A complicated diffraction problem must be solved to find the total value of the reflection coefficient at the open end of a waveguide with a flow. We restrict ourselves to the much simpler problem of finding a solution when the flow influence is small and $|\Delta R| \ll 1$. This approximation may be valid if the flow is far enough from the channel

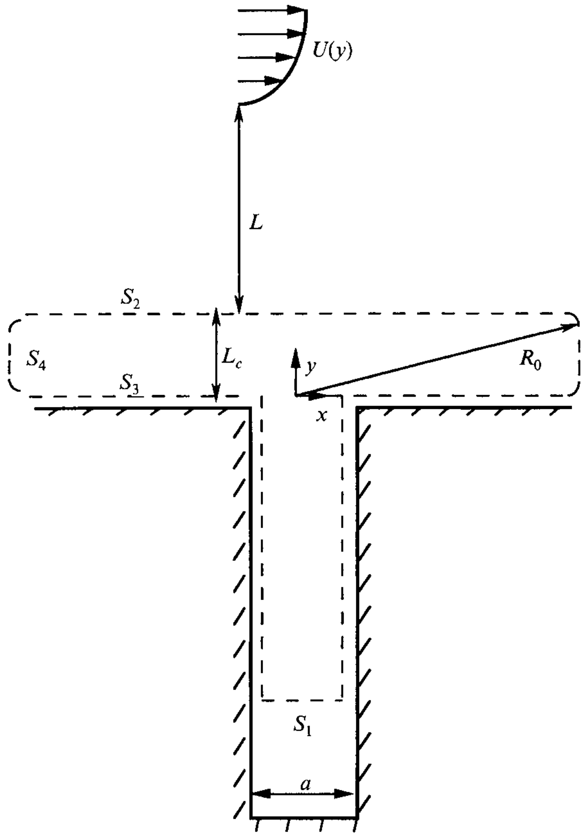

FIGURE 2. Shear flow near the open end of a shallow-water waveguide. The dashed line marks the integration path.

or if the flow is a very narrow jet. In both cases we may calculate the excitation of a waveguide by surface wave perturbations penetrating from the open end of the channel into the outer ocean and partially reflected from the flow, considering those reflected perturbations as an external force and neglecting multiple reflection between the flow and the waveguide.

To find the approximate reflection coefficient we use the integral relation

$$\oint_S \left(\eta^{(0)*} \frac{\partial \tilde{\eta}}{\partial n} - \tilde{\eta} \frac{\partial \eta^{(0)*}}{\partial n} \right) \mathrm{d}s = 0, \tag{8}$$

confined to the surface elevation field $\eta^{(0)}(\omega, \boldsymbol{r})$ without a shear flow and a perturbation $\tilde{\eta}(\omega, \boldsymbol{r})$ reflected by a flow. The asterisk in (8) stands for complex conjugate and the integral is along a surface S closing a domain with no stationary flow inside. The relation (8) follows from Green's formula for (3). Note that for two-dimensional fields the surface S essentially represents a contour in the (x, y)-plane.

We choose this contour, as shown in figure 2, made up of four different parts. The first one is a contour S_1 across the waveguide far enough from its open end. The second contour S_2 is a straight line dividing the harbour and the shear flow. It runs parallel to the shore (and to a shear flow at a distance L from it) outside the channel between its entrance and the flow. As this contour is supposed to go into an area with no steady flows we assume that it is close enough to the shore at a distance $L_c \ll L$. The third contour S_3 runs along the shore. Finally the contour S_4 is a closing arc with radius $R_0 \to \infty$. Integrating along the whole contour S one may connect the solution (6) inside the channel with a wave field outside.

We consider the shore line as a rigid boundary for shallow-water waves. A boundary condition $\partial\eta/\partial n = 0$ must be fulfilled there and so the integral (8) along the contour S_3 equals zero. This integral also vanishes at a closing contour S_4 due to the Sommerfeld radiation condition. Thus the sum of integrals along S_1 and S_2 equals to zero.

The value of the integral (8) along the waveguide cross-section S_1 may be found from (6) which represents the sum of $\eta^{(0)} + \tilde{\eta}$. Taking into account (7) we have

$$\int_{S_1} \left(\eta^{(0)*} \frac{\partial \tilde{\eta}}{\partial y} - \tilde{\eta} \frac{\partial \eta^{(0)*}}{\partial y} \right) \mathrm{d}x = -2\mathrm{i} \frac{\omega a}{c} H^2 \frac{|v_0|^2}{c^2} \Delta R. \tag{9}$$

Finally we express the integral along the dividing contour S_2 in terms of Fourier transforms of $\eta^{(0)}$ and $\tilde{\eta}$:

$$\left.\begin{aligned} \eta^{(0)} &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} H_k^{(0)} \, \mathrm{e}^{-\kappa y + \mathrm{i}kx} \, \mathrm{d}k, \\ \tilde{\eta} &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} (\tilde{G}_k \, \mathrm{e}^{+\kappa y} + \tilde{H}_k \, \mathrm{e}^{-\kappa y}) \, \mathrm{e}^{\mathrm{i}kx} \, \mathrm{d}k, \end{aligned}\right\} \tag{10}$$

where $\kappa = (k^2 - \omega^2/c^2)^{1/2}$ and $\mathrm{Re}\,\kappa > 0$. Substitution (10) into (8) gives us the integral along S_2 in the following form:

$$\int_{S_2} \left(\eta^{(0)*} \frac{\partial \tilde{\eta}}{\partial y} - \tilde{\eta} \frac{\partial \eta^{(0)*}}{\partial y} \right)_{y=L_c} \mathrm{d}x = \frac{1}{\pi} \int_{-\infty}^{+\infty} \kappa H_k^{(0)*} \tilde{G}_k \, \mathrm{d}k. \tag{11}$$

Now equating (9) and (11) we obtain the flow correction to the channel wave reflection coefficient

$$\Delta R = \frac{\mathrm{i}c^3}{2\pi a \omega H^2 |v_0|^2} \int_{-\infty}^{+\infty} \kappa H_k^{(0)*} \tilde{G}_k \, \mathrm{d}k. \tag{12}$$

Thus, we have expressed the reflection coefficient in terms of Fourier amplitudes of surface elevation. The next step is to connect the near-shore oscillation field with the shear flow influence.

The amplitudes $H_k^{(0)} + \tilde{H}_k$ of surface elevation harmonics incident on a shear flow and the amplitudes $\tilde{G}_k$ of reflected harmonics are connected by the reflection coefficient

$$r(\omega, k) = \frac{\tilde{G}_k}{H_k^{(0)} + \tilde{H}_k}. \tag{13}$$

An important approximation that we use below is based on a small value of the reflection coefficient for perturbations reflecting from the flow. This also results in a small value of ΔR in (6). To a first-order approximation we may write

$$\tilde{G}_k = r H_k^{(0)}. \tag{14}$$

Thus, using this approximation, we take into account only one-step wave scattering at a flow, neglecting reverberation – multiple reflection between the flow and the coast.

We also use the connection between velocity and surface elevation amplitudes,

$$H_k^{(0)} = \frac{-\mathrm{i}\omega H}{\kappa c^2} v_k^{(0)}, \tag{15}$$

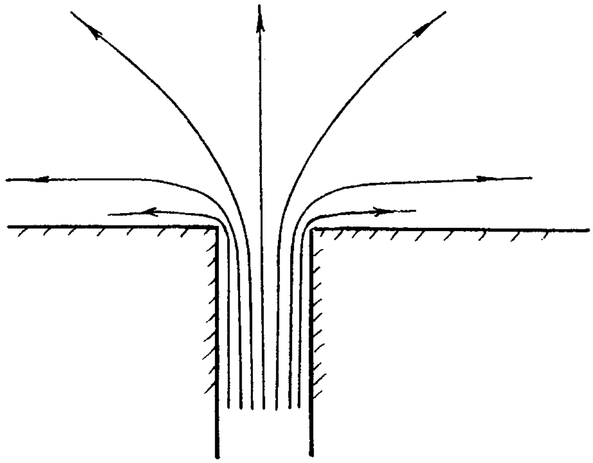

FIGURE 3. Streamlines of a potential flow from a channel open end.

that follows from (4). Here $v_k^{(0)}$ is the amplitude of the y-component of the velocity at the level $y = L_c$ taken to a zero-order approximation when the flow influence is neglected.

Substituting (14) and (15) into (12) we have

$$\Delta R = 4\mathrm{i}\frac{\omega a}{c}\int_{-\infty}^{+\infty} Wr(\omega, k)\frac{\mathrm{d}k}{2\pi\kappa}. \tag{16}$$

Here $W = |v_k^{(0)}/2v_0 a|^2$ is the power spectrum of the y-component of the velocity at $y = 0$ for some potential two-dimensional flow of inviscid incompressible fluid. It is the flow from a channel with unit cross-section if the values of the velocity inside the channel is unity (see figure 3). This field has a complicated structure (see Lamb 1932) but at distances $r \geqslant L_c \gg a$ it is an isotropic flow whose radial and angular components are

$$v_r = \frac{1}{\pi r}, \quad v_\phi = 0, \tag{17}$$

where r and ϕ are polar coordinates. The y-component of the velocity is

$$v = \frac{y}{\pi(x^2+y^2)} \tag{18}$$

for this flow and so to a first approximation with respect to the small parameter a/L we may use the formula

$$W = \left|\lim_{s\to 0}\int_{-\infty}^{+\infty}\frac{s}{\pi(x^2+s^2)}\mathrm{e}^{-ikx}\,\mathrm{d}x\right|^2 = 1. \tag{19}$$

3. Reflection from the flow

Formula (16) connects the waveguide fundamental wave reflection coefficient at the open end with a plane wave reflection coefficient $r(\omega, k)$ at a shear flow. To find the latter coefficient we consider the reflection problem of a plane wave incident on a shear flow which is located in the layer $|y-L| \leqslant l$. For small-amplitude shallow-water waves in a flow with the velocity profile $V = (U(y), 0, 0)$ we may derive from (1) the following linearized equations:

$$\left.\begin{aligned} &\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}+v\frac{\partial U}{\partial x}+g\frac{\partial \eta}{\partial x}=0,\\ &\frac{\partial v}{\partial t}+U\frac{\partial v}{\partial x}+g\frac{\partial \eta}{\partial y}=0,\\ &\frac{\partial \eta}{\partial t}+H\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0.\end{aligned}\right\} \tag{20}$$

If the plane wave comes from the half-plane $(y-L) < 0$, we have the following boundary conditions at a distance which is large in comparison with the flow size l:

$$\left.\begin{aligned} \eta\left(\frac{y-L}{l}\rightarrow-\infty\right) &= \eta_0\,\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}kx}\,(\mathrm{e}^{iky}+r\,\mathrm{e}^{-\mathrm{i}\kappa y}),\\ \eta\left(\frac{y-L}{l}\rightarrow+\infty\right) &= \eta_0\,\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}kx}q\,\mathrm{e}^{i\kappa y}.\end{aligned}\right\} \tag{21}$$

We have, therefore, a boundary value problem (20), (21) that is the same as for sound waves reflecting from a shear flow. So we may use any of the solutions already found for aeroacoustical problems of this type. Take, for example, the pioneering Miles–Ribner results obtained for the problem of sound reflection at a vortex sheet (Miles 1957*a*; Ribner 1957). If $U/c \ll 0$ and $k \gg \omega/c$ we may put $\kappa = (k^2)^{1/2}$ in the reflection coefficient. The Miles–Ribner reflection coefficient for a wave incident on vortex sheet located at $y = L$ may be written in this approximation as

$$r = \frac{(\omega-kU_0)^2-\omega^2}{(\omega-kU_0)^2+\omega^2}\mathrm{e}^{-2\kappa L}. \tag{22}$$

For a 'mono-speed' jet comprising two vortex sheets with the velocity profile

$$U(y) = \begin{cases} U_0 & \text{if} \quad |y-L| \leqslant l/2 \\ 0 & \text{if} \quad |y-L| > l/2, \end{cases} \tag{23}$$

the reflection coefficient is, in the same approximation,

$$r = \frac{[(\omega-kU_0)^4-\omega^4]\tanh(kl/2)}{2\omega^2(\omega-kU_0)^2+[(\omega-kU_0)^4+\omega^4]\tanh(kl/2)}\mathrm{e}^{-2\kappa L}. \tag{24}$$

There are also some analytical expressions for the coefficient $r(\omega, k)$ which have been found for other shear flows (Graham & Graham 1969). Those flows which have distributed vorticity may determine a complex reflection coefficient that corresponds to plane wave amplification or dissipation (Fabrikant 1976; Andronov & Fabrikant 1980*a*). Then integrating (16) along the wavenumber spectrum of the near field around the harbour entrance we may find the total wave energy input where different spatial harmonics contribute. This amplification mechanism originates from a wave–flow interaction in critical layers. But this is a relatively weak generation mechanism as only a small part of the flow (critical layer) participates in the interaction with the wave.

For resonator modes a stronger interaction may be realized. The mechanism of such an interaction is based on the connection of a waveguide mode with the flow as a whole, i.e. with flow oscillatory eigenmodes. Note that there are no critical layers for

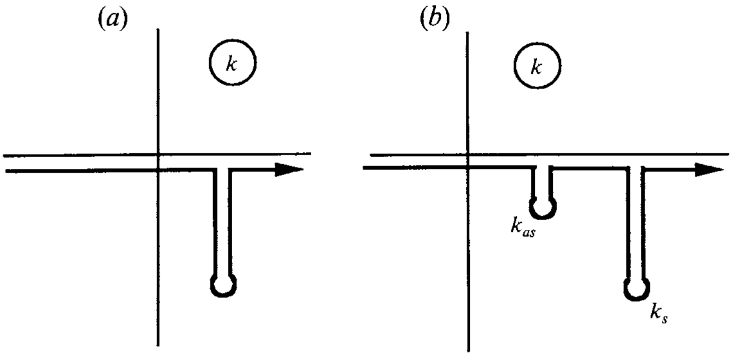

FIGURE 4. Integration contours for the reflection coefficients: (*a*) vortex sheet, (*b*) jet flow (k_s is a pole for the symmetric wave mode of the jet and k_{as} is a pole for the antisymmetric mode).

a vortex sheet or jet with a singular vorticity field and so the reflection coefficient of a plane wave does not contain any imaginary part $\operatorname{Im} r(\omega, k) = 0$. However, the interaction mechanism with flow eigenmodes still works for these flows. Below we analyse (16) to understand how this mechanism works.

The spectrum of eigenmodes in a shear flow is determined by the poles of the reflection coefficient $r(\omega, k)$. As a rule, natural oscillations of an inviscid flow are not neutral but unstable.† Then, for real-ω, poles of the reflection coefficient are located away from the real axis in the complex plane k. That is why the integral (16) should be taken not only along the real axis but probably along some more complicated path in the complex plane also.

The proper integration path may be found from the initial value problem that takes into account a 'switch-on' process for the waveguide oscillation mode. We use the Laplace method and investigate complex values of ω that are high in the upper half-plane of the complex frequency $\operatorname{Im} \omega > 0$. In that case all poles of the integrand in (16) are located in the upper half-plane of complex k, that is $\operatorname{Im} k > 0$. Accordingly, the integration in (16) may then be carried out along the real axis of k.

When the solution is continued analytically into the lower half-plane of complex ω, the poles of the integrand in (16) move in the complex k-plane towards the real axis and may intersect it. To be sure that formula (16) is an analytical continuation of the function ΔR corresponding to large positive $\operatorname{Im} \omega$, we have to deform the integration path in such a way as to leave poles above it. Also, the path must not intersect the cuts of the function $\kappa(k)$ specified by the condition $\operatorname{Re} \kappa > 0$ which guarantees that η and v will be finite as $y \rightarrow +\infty$. As a result we obtain a path consisting of the real axis of k supplemented by loops bypassing all the poles of the integrand in the lower complex half-plane of k – see figure 4.

The same path could obviously be obtained for a real-valued ω if we proceeded from boundary conditions at $x \rightarrow \pm\infty$. For a flow from $x = -\infty$ to $x = +\infty$ surface elevation and velocity perturbations must decay as $x = -\infty$ but may grow as $x = +\infty$ due to the flow instability. Functions of that kind are represented by integrals in the complex k-plane along contours of the indicated type. Poles of $r(\omega, k)$ which are inside the path loops in figure 4 correspond to unstable natural modes of the flow.

These considerations are exactly analogous to those which have been used to derive the Lin bypass rule in the theory of hydrodynamic instability of an ideal fluid (Case 1960) and to derive the Landau bypass rule in plasma wave theory (Landau 1965). The

† A comprehensive investigation of the vortex sheet eigenmodes in a compressible fluid is contained in Miles (1958) and Landau & Lifshits (1987).

only difference lies in the fact that a displacement of the integration path in these situations is, as a rule, very slight and simply corresponds to the rule of bypassing a pole on the real axis. In our situation, on the other hand, we find below that the displacement of the integration path may appear to be considerable because it is sometimes necessary to bypass poles far from the real axis. Note that such a path of integration in the complex k-plane is involved in studies of sources radiating near a vortex sheet (Jones & Morgan 1972).

4. Harbour self-excitation

Formula (16) with the integration path discussed above gives us to a first approximation the flow-induced correction ΔR to the reflection coefficient of the waveguide fundamental mode from an open end. The reflection determines damping or growth for oscillations in a resonator incorporating the waveguide as a constituent part (figure 1). To find the oscillation growth rate for a rectangular harbour (figure 1 *a*) we proceed from wave energy considerations for a fundamental mode in a waveguide.

The density of wave energy per unit area for a shallow-water wave may be written in the form (see Mei 1989)

$$\epsilon = \rho g \langle \eta^2 \rangle, \tag{25}$$

where angle brackets denote averaging over the wave period. Substituting the fundamental wave solution (6) into (25) we find the total wave energy in a channel, which is the sum of the energies of two counterpropagating waves:

$$E = \tfrac{1}{2}\rho g a L_0 |\eta_0|^2 (1 + |R + \Delta R|^2), \tag{26}$$

where L_0 is the channel length.

The wave energy flux for a fundamental wave mode propagating with a speed c is

$$S = ac\epsilon. \tag{27}$$

The difference between wave energy fluxes for a wave reflected from the channel entrance and for an incident wave is therefore

$$\Delta S = \tfrac{1}{2} ac\rho g |\eta_0|^2 (|R + \Delta R|^2 - 1). \tag{28}$$

Assuming that the undisturbed reflection coefficient $R = 1$ we can find to a first approximation the resonator oscillations growth rate in the form

$$\gamma = \frac{\Delta S}{2E} = \frac{\omega}{4\pi n} \operatorname{Re} \Delta R, \tag{29}$$

where n is the number of wavelengths that fit into the channel.

Thus a flow amplifies the reflected wave if $\gamma > 0$, i.e.

$$\operatorname{Re} \Delta R > 0. \tag{30}$$

Amplification of the reflected wave by a flow may cause excitation of harbour oscillations. Substituting (16) into (29) we may write the oscillation growth rate in the form

$$\gamma = -\frac{\omega^2 a}{\pi n c} \operatorname{Im} \int_{-\infty}^{+\infty} W r(\omega, k) \frac{\mathrm{d}k}{2\pi\kappa}, \tag{31}$$

where the integration must be along a path bypassing all poles of the coefficient $r(\omega, k)$ in the lower half-plane of complex k as shown in figure 4.

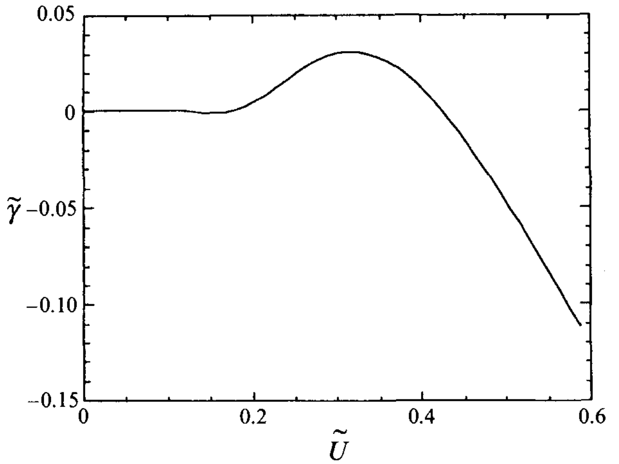

FIGURE 5. Growth rate $\tilde{\gamma} = \gamma/\gamma_{max}$ of a harbour oscillations generated by a vortex sheet as a function of the velocity $\tilde{U} = U_0/2\omega L$. Here $\gamma_{max} = \omega^2 a/(\pi nc\sqrt{2})$.

We shall analyse here the growth rate γ for a vortex sheet and a slender jet with piecewise velocity. The plane wave reflection coefficient $r(\omega, k)$ is real on the real k-axis for these simple flows with 'mono-speed' velocity profiles. Therefore, integration along the real axis in (31) does not contribute to the growth rate. The latter is determined solely by the residues at poles on the lower half-plane of complex k which correspond to unstable flow eigenmodes.

It has been shown by Andronov & Fabrikant (1980*a*) that a small but finite width of a shear layer induces a small imaginary component $\operatorname{Im} r(\omega, k)$ in the reflection coefficient for real values of ω and k and also slightly shifts the poles of $r(\omega, k)$. These effects create small variations of the growth rate, which we ignore here.

For a vortex sheet the plane wave reflection coefficient (22) has only one pole,

$$k = k_1 = (1-\mathrm{i})\,\omega/U_0, \tag{32}$$

in the lower half-plane of k (Betchov & Criminale 1967). Consequently, drawing the integrating path as in figure 4(*a*) and invoking (19) we obtain

$$\gamma = -\frac{\omega^2 a}{\pi nc\sqrt{2}} \mathrm{e}^{-2\omega L/U_0} \sin\left(\frac{2\omega L}{U_0} + \frac{\pi}{4}\right). \tag{33}$$

It is evident from (33) that the sign and magnitude of the oscillation growth rate in the channel are determined by the value of the parameter $\omega L/U_0$. The sign alternation of the function γ arises from the nature of the flow–harbour feedback. The open end of the channel excites a flow disturbance corresponding to Kelvin–Helmholtz instability of the vortex sheet. This disturbance in turn forces harbour oscillations if it is in the appropriate phase. The phase of the flow disturbance $2\omega L/U_0$ depends on the distance from the vortex sheet because the flow perturbation wavenumber (32) has an imaginary component for real ω.

The growth rate dependence on the vortex sheet velocity is shown in figure 5. We can see here that the growth rate reaches the maximum value

$$\gamma_{max} = 0.03 \frac{\omega^2 a}{\pi nc\sqrt{2}} \tag{34}$$

when the flow velocity $U_0 = 0.3 \times 2\omega L$.

We now consider another shear flow that yields a small value of ΔR so that the approximate expression (14) may be used. It is a slender jet (of width l) with the velocity profile (23). The reflection coefficient $r(\omega, k)$ for this jet (see (24)) has two poles in the lower half-plane of the complex variable k, corresponding to the symmetric and antisymmetric natural modes of the jet. The contour of integration in (31) for this flow is shown in figure 4(*b*). Assuming the small parameter $\omega L/U_0 \ll 1$ we can find wavenumbers corresponding to these poles (Betchov & Criminale 1971):

$$\left.\begin{aligned} k_s &= \frac{\omega}{U_0}\left[1-\mathrm{i}\left(\frac{\omega l}{4U_0}\right)^{1/2}\right], \\ k_{as} &= \frac{\sqrt{3}-\mathrm{i}}{2}\,\frac{\omega}{U_0}\left(\frac{4U_0}{\omega l}\right)^{1/3}. \end{aligned}\right\} \tag{35}$$

If the parameter $(2\omega L/U_0)(4U_0/\omega l)^{1/3} \gg 1$, the contribution of the pole k_{as} to the integral (31) is exponentially small so the latter integral is determined solely by the symmetric mode of a slender jet. Then the growth rate for harbour oscillations equals

$$\gamma = \frac{\omega^2 a}{4\pi nc}\,\frac{\omega^2 l}{U_0^2}\,\frac{\partial}{\partial k}(W\mathrm{e}^{-2kL})_{k=\omega/U_0} \tag{36}$$

to a first approximation with respect to the small parameter $\omega l/U_0 \ll 1$.

We can see from (36) that contributions to the growth rate of harbour oscillations are provided by jet perturbation harmonics that are synchronous with the jet velocity. Note that the spectrum shape of the velocity oscillations at the open end is important here. The growth rate depends on whether the spectrum increases or decreases at the synchronous wavenumber $k_0 = \omega/U_0$. If the spectrum increases with k, then the jet amplifies the oscillations, giving energy to them.

The latter result has a simple explanation: if the flow goes through the localized near field of a monopole oscillating source, that is an open end of a channel, the flow particles interact both with oscillation harmonics that lag the flow, thus slowing it, and with harmonics which move faster than the flow, thus accelerating the flow particles. If the power spectrum increases with k the slender jet interacts more strongly with oscillation harmonics that lag it, as those harmonics have a larger power spectrum.

It is likely that the derivative of the oscillation spatial spectrum has the same role here as the second derivative of the velocity profile in Miles' theory of wind-wave generation (Miles 1957*b*). Wind waves are amplified by a shear flow when the second derivative of the velocity is negative. There are more particles moving faster than a wave in this case and so the wave takes more energy from these particles than it gives energy to lagging particles (see more detailed explanation in Andronov & Fabrikant 1980*a*).

Analysing (36) we obtain the growth rate for a far jet when $L \gg a$:

$$\gamma = -\frac{\omega}{4\pi n}\,\frac{\omega a}{c}\,\frac{\omega^2 lL}{U_0^2}\,\mathrm{e}^{-2\omega L/U_0} < 0. \tag{37}$$

In this case the slender jet attenuates harbour oscillations.

To find the growth rate for a jet close enough to the harbour entrance we have to know the velocity power spectrum $W(k)$. The explicit form of the oscillation velocity distribution just near the channel entrance is unknown. It is reasonably clear, however, that the space scale of the variation of the oscillatory velocity along the x-axis is the width of the channel a. Consequently, the characteristic parameter governing the

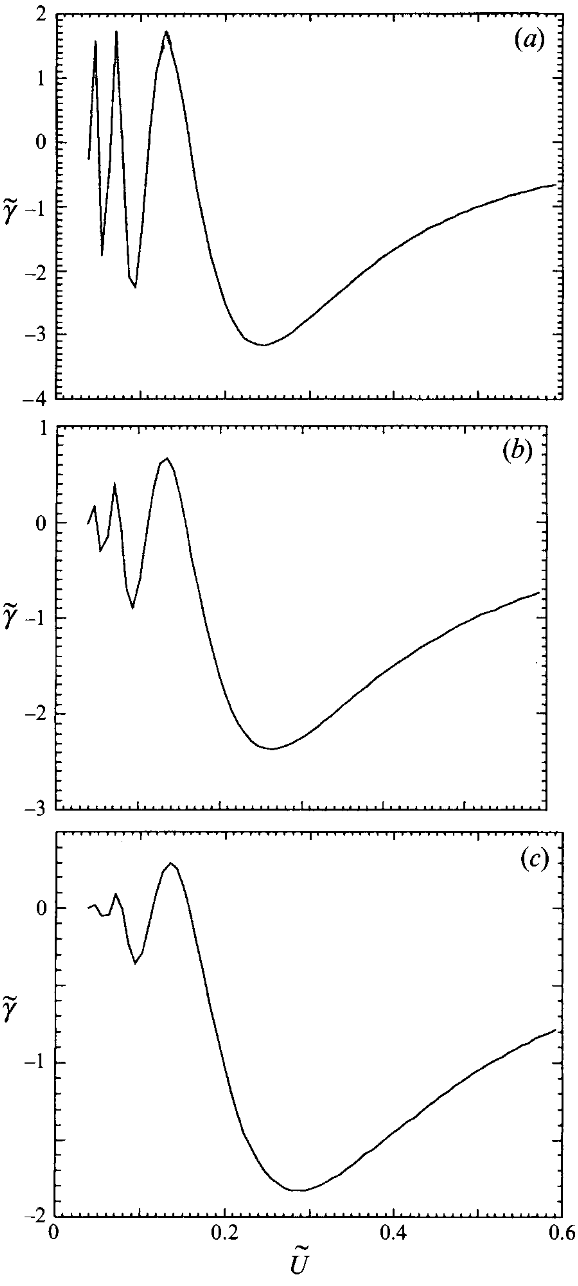

FIGURE 6. Growth rate $\tilde{\gamma} = \gamma/\gamma_{max}$ of harbour oscillations generated by a jet flow versus the jet velocity $\tilde{U} = U_0/\omega a$. Here $\gamma_{max} = \omega^2 l/(4\pi nc)$. (a) $L = 0$, (b) $2L/a = 0.1$, (c) $2L/a = 0.3$.

excitation must be the transit angle $\theta = \omega a/U_0$. This parameter is a measure of a wave phase change for the jet natural wave mode during its propagation past the harbour entrance. In particular, if we adopt the following elementary model for the oscillatory velocity amplitude:

$$v(y=0) = \mathrm{e}^{-\mathrm{i}\omega t}\begin{cases} 2v_0 & \text{for} \quad |x| < a/2 \\ 0 & \text{for} \quad |x| \geqslant a/2, \end{cases} \tag{38}$$

then the spectrum derivative $(\partial W/\partial k)_{k=\omega/U_0}$ and the growth rate

$$\gamma = \frac{\omega^2 l}{4\pi nc}\left[\theta^2 \frac{\partial}{\partial\theta}\left(\frac{4\sin^2(\theta/2)}{\theta^2}\mathrm{e}^{-2\theta L/a}\right)\right]_{\theta=\omega a/U_0} \tag{39}$$

oscillate with the variation of θ, changing sign as θ varies by an amount of order π – see figure 6.

For large values of the jet velocity, when $\theta \geqslant 7$, oscillations are attenuated by the jet. The growth rate has some local maxima for smaller values of the parameter θ, but their value decreases rapidly as the distance L grows.

Note here that, as the growth rate is an oscillating function of the harbour size and the jet speed, it appears very sensitive to the jet and harbour parameters. That may result in a strong variability of the harbour oscillation levels.

An estimation of the growth rate for a harbour with depth $H = 20$ m and entrance width $a = 300$ m if the jet scale and harbour–flow distance are $l/2 = L = 15$ m shows that harbour oscillations with period $T = 2\pi/\omega = 5$ min begin to grow for flow speed $U_0 \approx 0.8$ m s^{-1}. The value of the growth rate is of order $\gamma \approx 10^{-4}$ s^{-1}. Accordingly, the time of oscillation growth is about 2.7 hours.

5. Conclusions

The mechanism of instability described above is different from any of mechanisms considered previously for harbour excitation. The harbour oscillations arise as a result of linear instability in a system of coupled surface-wave resonator and shear flow.

The relation between this mechanism and the external forcing mechanisms studied before is the same as the relation between the two mechanisms of wind wave generation found by Miles (1957*b*) and Phillips (1977). The Miles' mechanism is based on wave amplification due to their interaction with a flow while the Phillips' mechanism is simply wave generation by an external oscillating force. The generation of harbour oscillations described here is also based on wave–flow interaction resulting in oscillation instability. But the physical nature of instability is different from the Miles' mechanism: it is not resonance wave interaction with a flow in a critical layer. Here harbour oscillations interact with the flow as a whole, not only with a narrow critical layer. This instability is based on a feedback mechanism confining harbour natural modes and eigenmodes of the flow. Oscillating velocity around the open end of a harbour excites natural wave modes in a flow. These waves propagate along the flow going through the near field of the channel entrance. They carry both velocity and surface-elevation oscillating perturbations with phase shifting along the distance. If the wave phase shift, arising during wave propagation through the near field, has an appropriate value, the surface-elevation perturbation excites the harbour entrance thus closing the feedback loop.

Note that a similar mechanism has already been proposed to explain the excitation of sound in whistles, flutes and organ pipes (Coltman 1968; Elder 1978; Andronov & Fabrikant 1980*b*).

An important question arising from the analysis of the linear instability is the problem of nonlinear saturation of that instability. There are several mechanisms of nonlinear dissipation for shallow-water waves inside a harbour, for example wave breaking on a shore or turbulent dissipation in the bottom boundary layer. Now we estimate the nonlinear saturation for one of the most important mechanisms: energy loss due to separation of the oscillating flow at the channel entrance (Mei 1989). We use here the hydraulic formula for a surface-elevation jump in a quasi-static flow through a slot:

$$\eta = \frac{f}{2g} v|v|, \tag{40}$$

where the dimensionless empirical coefficient f depends on the geometrical form of the entrance and is of order unity. It allows us to find an energy loss averaged over a wave period:

$$Q = -\frac{\mathrm{d}E}{\mathrm{d}t} = \left\langle a \int_0^{H+\eta} \rho g z \,\mathrm{d}z v \right\rangle = 2Ha\rho g \frac{f}{2g} \langle v^2|v| \rangle. \tag{41}$$

For sinusoidal velocity oscillations we have from (41)

$$Q = \frac{8}{3\pi} \rho v_0^3 f a. \tag{42}$$

In the equilibrium state the value of the energy loss equals the gain from instability so that

$$Q = \gamma E. \tag{43}$$

Now we may substitute into (43) the formula (42) and the expression (26) for the total wave energy E. Assuming the value $f = 1$ and taking into account the connection (7) between amplitudes of velocity and elevation in the fundamental wave mode we find the amplitude of the surface elevation in the form

$$\eta_0 = \frac{3\pi}{4} \frac{HL_0}{c} \gamma, \tag{44}$$

where the growth rate γ can be found from (33) or (39) and the oscillations are considered on the quarter-wave mode ($n = \frac{1}{2}$). For a jet flow with the parameters discussed above we have from (44) the value of the surface elevation $\eta_0 = 0.8$ m and the corresponding value of the velocity amplitude in the harbour entrance is 0.5 m s^{-1}. These values are comparable with amplitudes of resonant oscillations excited by external forces in harbours. Very strong incident waves like, for example, tsunamis may excite larger resonant oscillations. But the flow excitation mechanism may be responsible for the occurrence of harbour oscillations in the absence of any waves incident on the harbour entrance.

Note in conclusion that the evaluations made above are very rough, showing only typical orders of the evaluated parameters. More precise calculations appropriate for engineering applications must include consideration of a more realistic flow profile as well as the complicated topography of a real harbour which affects the eigenmode frequency and space amplitude distribution. As for the physical nature of harbour oscillations excited by shear flows, there is also another excitation mechanism which has not been considered yet: external waves interacting with a coastal flow could produce intensive oscillations in a harbour. That is, however, a more complex problem which deserves separate consideration.

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REFERENCES

Abrol, V. 1990 Numerical modelling of tidal calculation and tide-induces water level variation in Bombay harbour. *Indian J. Mar. Sci.* **19**, 89–94.

ANDRONOV, A. A. & FABRIKANT, A. L. 1980*a* Theory of aerodynamic self-excitation of sound: amplification of surface waves. *Sov. Phys. Acoust.* **26**, 370–374.

ANDRONOV, A. A. & FABRIKANT, A. L. 1980*b* Theory of aerodynamic self-excitation of sound: resonator whistles. *Sov. Phys. Acoust.* **26**, 461–464.

BARNETT, J. P. & KENYON, K. E. 1975 Recent advantages in the study of wind waves. *Rep. Prog. Phys.* **38**, 667–729.

BETCHOV, R. & CRIMINALE, W. O. 1967 *Stability of Parallel Flows.* Academic Press.

BOWERS, E. C. 1977 Harbour resonance due to set-down beneath wave groups. *J. Fluid Mech.* **79**, 71–92.

CASE, K. M. 1960 Hydrodynamic stability as a problem with initial data. *Phys. Fluids* **3**, 143–148.

COLTMAN, J. W. 1968 Sounding mechanism of the flute and organ pipes. *J. Acoust. Soc. Am.* **44**, 983–992.

ELDER, S. A. 1978 Edgetones versus pipetones. *J. Acoust. Soc. Am.* **64**, 1721–1723.

FABRIKANT, A. L. 1976 Resonant interaction of sound with a parallel shear flow. *Sov. Phys. Acoust.* **22**, 56–59.

GILMOUR, A. E. 1990 Response of Wellington harbour to the tsunamis of 1960 and 1964. *New Zealand J. Mar. Freshwater Res.* **24**, 229–231.

GRAHAM, B. B. & GRAHAM, E. B. 1969 Effect of a shear layer on plane waves of sound in a fluid. *J. Acoust. Soc. Am.* **46**, 169–175.

JONES, D. S. & MORGAN, J. D. 1972 Instability of the vortex sheet under the action of sound. *Proc. Camb. Phil. Soc.* **72**, 465–491.

LAMB, H. 1932 *Hydrodynamics*, 6th edn. Cambridge University Press.

LANDAU, L. D. 1965 *Collected Papers.* Gordon and Breach.

LANDAU, L. D. & LIFSHITS, E. M. 1987 *Fluid Dynamics.* Pergamon Press.

LIU, P. L.-F. 1989 A note on long waves induced by short-wave groups over a shelf. *J. Fluid Mech.* **205**, 163–170.

LIU, P. L.-F., DINGEMANS, M. W. & KOSTENSE, J. K. 1990 Long waves generation due to the refraction of short-wave groups over a shelf current. *J. Phys. Oceanogr.* **20**, 53–59.

MEI, C. C. 1989 *The Applied Dynamics of Ocean Surface Waves.* World Scientific.

MEI, C. C. & AGNON, Y. 1989 Long-period oscillations in a harbour induced by incident short waves. *J. Fluid Mech.* **208**, 595–608.

MEI, C. C. & BENMOUSSA, C. 1984 Long waves induced by short-wave groups over an uneven bottom. *J. Fluid Mech.* **139**, 219–235.

MILES, J. W. 1957*a* On the reflection of sound at an interface of relative motion. *J. Acoust. Soc. Am.* **29**, 226.

MILES, J. W. 1957*b* On the generation of surface waves by shear flows. *J. Fluid Mech.* **3**, 185–204.

MILES, J. W. 1958 On the disturbed motion of a plane vortex sheet. *J. Fluid Mech.* **4**, 538–552.

MILES, J. W. 1974 Harbor seiching. *Ann. Rev. Fluid Mech.* **6**, 17–35.

MILES, J. W. & MUNK, W. H. 1961 Harbor paradox. *Proc. ASCE, J. Waterways Harbor Div.* **87**, 111–130.

OKIHIRO, M., GUZA, R. T. & SEYMOUR, R. J. 1993 Excitation of seiche observed in a small harbor. *J. Geophys. Res.* **98**, 18201–18211.

PHILLIPS, O. M. 1977 *The Dynamics of the Upper Ocean.* Cambridge University Press.

RIBNER, H. S. 1957 Reflection, transmission and amplification of sound by a moving medium. *J. Acoust. Soc. Am.* **29**, 435.

WU, J.-K. & LIU, P. L.-F. 1990 Harbour excitations by incident wave groups. *J. Fluid Mech.* **217**, 595–613.

ZELT, J. A. & RAICHLEN, F. 1990 A lagrangian model for wave-induced harbour oscillations. *J. Fluid Mech.* **213**, 203–225.

ZHOU, C. & LIU, P. L.-F. 1987 Second-order low-frequency wave forces on a vertical cylinder. *J. Fluid Mech.* **175**, 143–155.